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Book review

'Theory of Solidification' by Stephen H. Davis, Cambridge University Press, 18 October 2001, ISBN 0-521-65080-1, $74.95, £ 50.

As its title implies, this monograph presents the continuum theory of liquid–solid phase transformations. The book is designed in a constructive fashion, where the first few chapters presents the fundamental building blocks of the theory, and subsequent chapters probe the subject more deeply, using increasingly sophisticated mathematics. The book is thus accessible to readers with varying training in analysis, and focuses on making the connection between mathematical theory and observations of microstructure development.

One might liken the treatment of the subject to a visit to a ski area. The first three chapters are suitable for novices with only modest skills. These chapters introduce the basics: thermodynamic equilibrium, capillarity, and kinetics for pure and binary substances. The concept of linear stability is introduced, showing how the interaction of capillarity and thermodynamics leads to spatial pattern selection.

The next three chapters are the "black diamonds" of the book, requiring considerable mathematical sophistication. Whereas the linear stability analysis introduced in the previous chapter applies to the formation of the initial patterns, nonlinear analysis is required to establish the patterns at finite size. These chapters present nonlinear analyses, as well as the important concepts of crystalline anisotropy and how kinetics leads to departures from equilibrium at high solidification rates.

Chapter 7 presents the theory of dendritic growth. It begins with the analysis of Ivantsov, who showed that a solid paraboloid of revolution growing into an undercooled melt can satisfy the energy transport equation while propagating at constant velocity V and with unvarying shape (characterized by the tip radius R). This leads to a conundrum. Whereas in experiments, dendrites always select a unique pair of V and R as the so-called "operating state", the transport solution is satisfied for any state where the product VR is a constant depending on the undercooling. Subsequent selection theories are then presented, including more physical phenomena, such as surface tension and its anisotropy, and more sophisticated analytical techniques. The chapter then goes on to describe sidebranch formation, and solidification of arrays of dendrites. This subject is one where Professor Davis has not worked extensively, and the chapter does not provide quite the same crisp delineation of competing theories that one finds in the others.

Chapter 8 considers eutectic growth. It begins with the analysis of Jackson and Hunt, who developed the first theory to predict the observed relationship between eutectic spacing λ and growth speed V, $\lambda^2 V = \text{constant}$. It then takes up the subject of instabilities in growth front shape and lamellar spacing.

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Fluid flow in the melt is introduced in Chapters 9 and 10. Flow affects the transport by advection, but it also introduces new possibilities for instabilities. The topic is logically divided into microscale effects, treated in Chapter 9, and macroscopic effects, examined in Chapter 10. Some of analysis is difficult, while others can be followed by ‘‘advanced intermediates’’. Professor Davis has made numerous important contributions in this area, and the presentation here reflects that. In Chapter 9, several prototype flows are first introduced, and then the effect of these flows on morphological stability is considered. Chapter 10 takes a similar approach, but considers instead interactions of flow with structure on the mesoscale. Thus double diffusive flows and their effect on the evolution of the two-phase mushy zone are considered. This topic provides a rich set of interesting problems, and the exposition is very clear and informative.

Up to this point, very little has been said about numerical methods. Recently, a lot of progress in understanding microstructures has been made through phase-field method simulations. The mathematics of the method is introduced in this chapter, and some results are included as well. This chapter provides an introduction to the topic, but researchers who want to perform such calculations will need to follow the references for a more complete treatment.

In summary, this monograph presents the continuum theory of solidification. Novices can obtain a good understanding of how the physics and mathematics combine to give pattern selection during solidification. More experienced practitioners can benefit from the higher level analyses, including the nonlinear analyses of directional solidification and the interaction of fluid flow with solidification.

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